10th Class 2020

Math (Science)	Group-II	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Write in the standard form of quadratic equation:

$$(x + 7)(x - 3) = -7$$

Ans Given:

$$(x + 7)(x - 3) = -7$$
$$x(x - 3) + 7(x - 3) = -7$$
$$x^2 - 3x + 7x - 21 = -7$$
$$x^2 + 4x - 21 + 7 = 0$$
$$x^2 + 4x - 14 = 0 \qquad (A)$$

Equation 'A' is the standard form of quadratic equation.

(ii) Solve equation by using quadratic formula:

$$2 - x^2 = 7x$$

Ans Given quadratic equation:

$$2 - x^2 = 7x$$

By arranging the above equation as standard form:

$$0 = 7x + x^2 - 2$$

$\Rightarrow x^2 + 7x - 2 = 0$

From above equation,

$$a = 1, b = 7, c = -2$$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{(7)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$= \frac{-7 \pm \sqrt{57}}{2}$$

The solution set will be:

$$\left\{\frac{-7 \pm \sqrt{57}}{2}\right\}.$$

(iii) **Solve by factorization: $5x^2 = 15x$.**

Ans

$$5x^2 - 15x = 0$$
$$5x(x - 3) = 0$$

From the above equation,

$$5x = 0 \qquad x - 3 = 0$$
$$\boxed{x = 0} \qquad \boxed{x = 3}$$

Thus, solution set: $\{0, 3\}$.

(iv) **Find the discriminant of quadratic equation:**

$$9x^2 - 30x + 25 = 0$$

Ans

$$9x^2 - 30x + 25 = 0$$
$$\text{Discriminant} = b^2 - 4ac$$

Here, from the above equation:

$$a = 9,\ b = -30,\ c = 25$$
$$\text{Discriminant} = (-30)^2 - 4(9)(25)$$
$$= 900 - 900$$
$$= 0$$

(v) **Use synthetic division to find the quotient and remainder, when:**

$$(x^3 + x^2 - 3x + 2) \div (x - 2)$$

Ans Given: $x - a = x - 2$

$$-a = -2$$
$$a = 2$$

Now, write the coefficients of dividend in a row and $a = 2$ on the left side.

$$\begin{array}{c|cccc} & 1 & 1 & -3 & 2 \\ 2 & \downarrow & 2 & 6 & 6 \\ \hline & 1 & 3 & 3 & \big| \ 8 \end{array}$$

$\therefore$ Quotient $= Q(x) = x^2 + 3x + 3$ and

Remainder = R = 8.

(vi) Discuss the nature of the roots of equation

$$16x^2 - 8x + 1 = 0$$

Ans Here, $a = 16, b = -8, c = 1$

Discriminant $= b^2 - 4ac$

$= (-8)^2 - 4(16)(1)$

$= 64 - 64$

$= 0$

So, the nature of the roots of the equation is rational and equal.

(vii) Find the third proportional to 6, 12.

Ans Let C be the third proportional, then

$6 : 12 :: 12 : C$

$\therefore$ Product of Extremes = Product of Means

$6 \times C = 12 \times 12$

$6C = 144$

$C = \frac{144}{6}$

$C = 24.$

(viii) Define inverse variation.

Ans If two quantities are related in such a way that when one quantity increases, the other decreases is called inverse variation.

(ix) Find a, if the ratios a + 3 : 7 + a and 4 : 5 are equal.

Ans Since the ratios $a + 3 : 7 + a$ and $4 : 5$ are equal.

$\therefore$ in fraction form

$$\frac{a+3}{7+a} = \frac{4}{5}$$

$5(a + 3) = 4(7 + a)$

$5a + 15 = 28 + 4a$

$5a - 4a = 28 - 15$

$a = 13$

Thus, the given ratios will be equal if $a = 13$.

3. Write short answers to any SIX (6) questions: (12)

(i) Define identity.

Ans An identity is an equation which is satisfied by all the values of the variables involved.

(ii) Resolve into partial fractions: $\dfrac{7x-9}{(x+1)(x-3)}$.

Ans Let $\dfrac{7x-9}{(x+1)(x-3)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-3)}$ (i)

Multiplying by $(x+1)(x-3)$, we get

$$7x - 9 = A(x-3) + B(x+1)$$
$$7x - 9 = Ax - 3A + Bx + B$$
$$7x - 9 = Ax + Bx - 3A + B$$
$$7x - 9 = (A+B)x - 3A + B$$

By comparing coefficients of x and constant terms,

$A + B = 7$ (ii)

$-3A + B = -9$ (iii)

By subtracting (iii) from (ii), it gives

$$\begin{array}{rcl} A + B &=& 7 \\ \mp 3A \pm B &=& \mp 9 \\ & & {}_{+\quad -\quad +} \\ \hline 4A &=& 16 \end{array}$$

$$A = \frac{16}{4}$$
$$A = 4$$

Put $A = 4$ in (ii)

$$4 + B = 7$$
$$B = 7 - 4$$
$$B = 3$$

By putting values of A, B in (i), we get

$$\frac{7x-9}{(x+1)(x-3)} = \frac{4}{(x+1)} + \frac{3}{(x+3)}.$$

(iii) Define function.

Ans Suppose A and B are two non-empty sets, then relation $f : A \rightarrow B$ is called a function if (i) Dom f = set A

(ii) $\forall$ x $\in$ A, we can associate some unique image element y = f(x) $\in$ B.

(iv) If X = Set of prime numbers less than or equal to 17, and Y = Set of first 12 natural numbers, then find $X \cap Y$.

Ans Here, X = {2, 3, 5, 7, 11, 13, 17}

Y = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

$X \cap Y = \{2, 3, 5, 7, 11, 13, 17\} \cap \{1, 2, 3, ----, 12\}$

$X \cap Y = \{2, 3, 5, 7, 11\}$.

(v) If A = {a, b} and B = {c, d}, then find B × A.

Ans $B \times A = \{c, d\} \times \{a, b\}$

$= \{(c, a), (c, b), (d, a), (d, b)\}$.

(vi) Find a and b if: (2a + 5, 3) = (7, b – 4).

Ans Here, (2a + 5, 3) = (7, b – 4)

$\Rightarrow$ 2a + 5 = 7 ; 3 = b – 4

2a = 7 – 5 ; 3 + 4 = b

2a = 2 ; $\Rightarrow$ b = 7

$a = \frac{2}{2}$

a = 1

So, a = 1 and b = 7.

(vii) Define class limits.

Ans The minimum and the maximum values defined for a class or group are called class limits.

(viii) The salaries of five teachers are as follows. Find mean salary: 11500, 12400, 15000, 14500, 14800.

Ans The maximum value:

$X_m = 15,000$

The minimum value:

$X_o = 11,500$

Range = $X_m - X_o$

= 15,000 – 11,500

= 3,500

(ix) Find harmonic mean for the data X = 12, 5, 8, 4.

Ans

X	$\frac{1}{X}$
12	0.0833
5	0.2
8	0.125
4	0.25
	0.6583

$$H.M = \frac{n}{\Sigma(\frac{1}{x})} = \frac{4}{0.6583}$$

$$\boxed{H.M = 6.0763}$$

4. Write short answers to any SIX (6) questions: (12)

(i) Define radian.

Ans The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

(ii) Convert $\frac{\pi}{4}$ radians to degree.

Ans $\frac{\pi}{4}$ radians $= \frac{1}{4}$ (π radians)

$$= \frac{1}{4}(180^\circ)$$

$$= 45^\circ$$

(iii) In a ΔABC, a = 17 cm, b = 15 cm, c = 8 cm, find m$\angle$B.

Ans In a ΔABC ; a = 17 cm, b = 15 cm and c = 8 cm.

Here, m $\angle$ B = ?

If it is a right angled triangle,

$$\sin m\angle B = \frac{b}{a}$$

$$\sin m\angle B = \frac{15}{17}$$

$$\sin m\angle B = 0.882$$

$m\angle B = \sin^{-1}(0.882)$

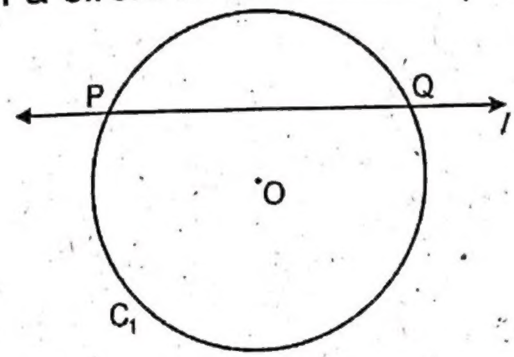
$m\angle B = 61.9^{\circ}$.

(iv) Define minor arc of a circle.

Ans An arc which is less than a semi-circle is called minor arc of a circle.

(v) Define secant of a circle.

Ans A secant is a straight line which cuts the circumference of a circle in two distinct points.



In the above figure, l indicates the secant line to the circle C_1.

(vi) Define an arc of a circle.

Ans A part of circumference of a circle is called an arc.

(vii) Define circum angle.

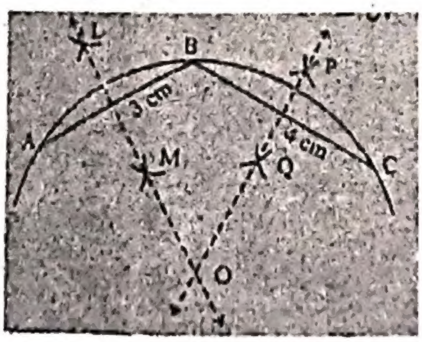
Ans A circum angle subtended between any two chords of a circle having common point on its circumference.

(viii) Define circumscribed circle.

Ans The circle passing through the vertices of the triangle is called circumscribed circle.

(ix) If $|\overline{AB}| = 3$ cm, $|\overline{BC}| = 4$ cm are the lengths of two chords of an arc, then locate the centre of the arc.

Ans



Steps:

1. Draw an arc $\widehat{ABC}$.
2. Draw $|\overline{AB}| = 3$ cm and $|\overline{BC}| = 4$ cm.
3. Draw $\overline{LM}$ and $\overline{PQ}$ right bisectors of $\overline{AB}$ and $\overline{BC}$, respectively. $\overline{LM}$ and $\overline{PQ}$ intersect at point O.
4. O is the required centre of an arc $\widehat{ABC}$.

(Part-II)

Note: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$$x^2 - 2x - 195 = 0$$

Ans Given: $x^2 - 2x - 195 = 0$ (i)

Shifting constant term -195 to the right, we have

$$x^2 - 2x = 195$$

Adding the square of $\frac{1}{2}$ X coefficient of x, that is

$$x^2 - 2x + (-1)^2 = 195 + (-1)^2$$

$$(x)^2 - 2(x)(1) + (-1)^2 = 195 + 1$$

$$(x - 1)^2 = 196$$

Taking square root of both sides of the above equation, we have

$$\sqrt{(x-1)^2} = \pm\sqrt{196}$$

$$x - 1 = \pm 14$$

$x - 1 = 14$; $x - 1 = -14$

$x = 14 + 1$; $x = -14 + 1$

$\boxed{x = 15}$; $\boxed{x = -13}$

Thus, the solution set is $\{15, -13\}$.

(b) Find the value of k, if roots of the equation are equal $x^2 + 2(k + 2)x + (3k + 4) = 0$. (4)

Ans $x^2 + 2(k + 2)x + (3k + 4) = 0$

Here $a = 1, b = 2(k + 2), c = 3k + 4$

As the roots are equal, so

$$\text{Discriminant} = 0$$
$$b^2 - 4ac = 0$$
$$[2(k + 2)]^2 - 4(1)(3k + 4) = 0$$
$$4(k^2 + 4k + 4) - 12k - 16 = 0$$
$$4k^2 + 16k + 16 - 12k - 16 = 0$$
$$4k^2 + 4k = 0$$
$$4k(k + 1) = 0$$
$$4k = 0 \quad ; \quad k + 1 = 0$$
$$k = \frac{0}{4} \quad ; \quad k = -1$$
$$k = 0$$

So, $k = 0$ and $k = -1$ are the values of k.

Q.6.(a) Using theorem of componendo-dividendo, solve the equation: (4)

$$\frac{\sqrt{x+3} + \sqrt{x-3}}{\sqrt{x+3} - \sqrt{x-3}} = \frac{4}{3}$$

Ans For Answer see Paper 2018 (Group-II), Q.6.(a).

(b) Resolve into partial fractions: (4)

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)}$$

Ans
$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)} = \frac{A}{(x + 2)} + \frac{B}{(x + 2)^2} + \frac{C}{(x + 3)} \quad \text{(i)}$$

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)} = \frac{A(x + 2)(x + 3) + B(x + 3) + C(x + 2)^2}{(x + 2)^2(x + 3)}$$

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)}(x + 2)^2(x + 3) = (x + 2)^2(x + 3)$$

$$= \frac{A(x+2)(x+3) + B(x+3) + C(x+2)^2}{(x+2)^2(x+3)}$$

$$x^2 + 7x + 11 = A(x+2)(x+3) + B(x+3) + C(x+2)^2 \quad \text{(ii)}$$

$$x^2 + 7x + 11 = A(x^2 + 5x + 6) + B(x+3) + C(x^2 + 4x + 4) \quad \text{(iii)}$$

For finding 'B':

Let $x + 2 = 0$

$x = -2$

By putting $x = -2$ in (iii), we get

$$(-2)^2 + 7(-2) + 11 = A((-2)^2 + 5(-2) + 6) + B(-2+3) + C((-2)^2 + 4(-2) + 4)$$

$$4 - 14 + 11 = A(4 - 10 + 6) + B(1) + C(4 - 8 + 4)$$

$$1 = 0 + B + 0$$

$$\boxed{B = 1}$$

For finding 'C':

Let $x + 3 = 0$

$x = -3$

By putting $x = -3$ in (iii), we get

$$(-3)^2 + 7(-3) + 11 = A((-3)^2 + 5(-3) + 6) + B(-3+3) + C((-3)^2 + 4(-3) + 4)$$

$$9 - 21 + 11 = A(9 - 15 + 6) + B(0) + C(9 - 12 + 4)$$

$$-1 = 0 + 0 + C(1)$$

$$\Rightarrow \quad \boxed{C = -1}$$

Compare coefficients of x^2 of (iii),

$$1 = A + C$$

As we have

$$C = -1$$

$$1 = A - 1$$

$$\Rightarrow \quad \boxed{A = 2}$$

Put the values of A, B, C in (i),

$$\frac{x^2 + 7x + 11}{(x+2)^2(x+3)} = \frac{2}{(x+2)} + \frac{1}{(x+2)^2} - \frac{1}{(x+3)}$$

Q.7.(a) If $U = \{1, 2, 3, 4, ---, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{1, 4, 7, 10\}$, then prove that $(A - B)' = A' \cup B$. (4)

Ans L.H.S $= (A - B)'$

$= U - (A - B)$

$= U - [\{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}]$

$= U - [\{3, 5, 9\}]$

$= \{1, 2, 3, 4, ----, 10\} - \{3, 5, 9\}$

$= \{1, 2, 4, 6, 7, 8, 10\}$ (i)

R.H.S $= A' \cup B$

$= (U - A) \cup B$

$= [\{1, 2, 3, 4, ----, 10\} - \{1, 3, 5, 7, 9] \cup B$

$= [\{2, 4, 6, 8, 10\}] \cup B$

$= \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\}$

$= \{1, 2, 4, 6, 7, 8, 10\}$ (ii)

From (i) and (ii), we have L.H.S = R.H.S

Hence proved.

(b) Find the variance about mean of the students, who obtained marks in Statistics: (4)

Marks y	62	62	65	68	67	48

Ans

Marks (y)	y^2
62	3844
62	3844
65	4225
68	4624
67	4489
48	2304
372	23330

$$\text{Variance} = S^2 = \frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2$$

$$S^2 = \frac{23330}{6} - \left(\frac{372}{6}\right)^2$$

$$= 3888.33 - (62)^2$$

$$= 3888.33 - 3844$$

$$\boxed{S^2 = 44.33}$$

Q.8.(a) Prove that: **(4)**

$$\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}=2\,\text{cosec}^2\,\theta$$

Ans L.H.S $=\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$

$$=\frac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$$

$$=\frac{2}{(1)^2-(\cos\theta)^2}$$

$$=\frac{2}{1-\cos^2\theta}$$

$$=\frac{2}{\sin^2\theta} \qquad [\because \ \sin^2\theta = 1-\cos^2\theta]$$

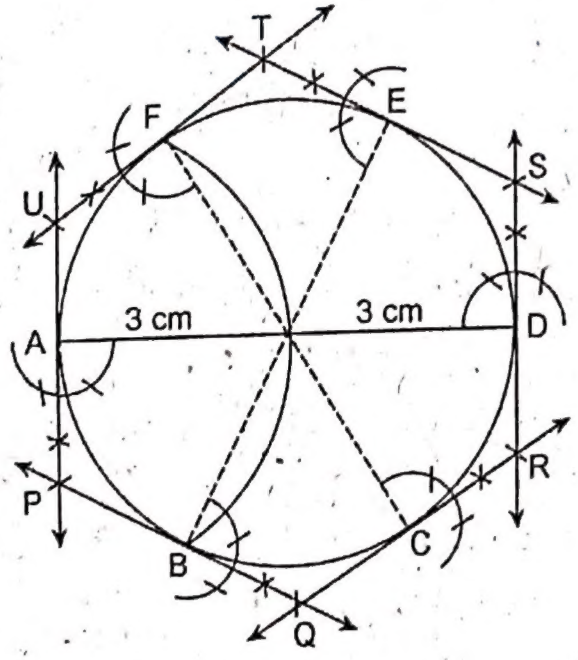
$$=2\left(\frac{1}{\sin^2\theta}\right)$$

$$=2\,\text{cosec}^2\,\theta \qquad [\because \ \frac{1}{\sin^2\theta}=\text{cosec}^2\,\theta]$$

= R.H.S, hence proved.

(b) Circumscribe a regular hexagon about a circle radius 3 cm. **(4)**

Ans

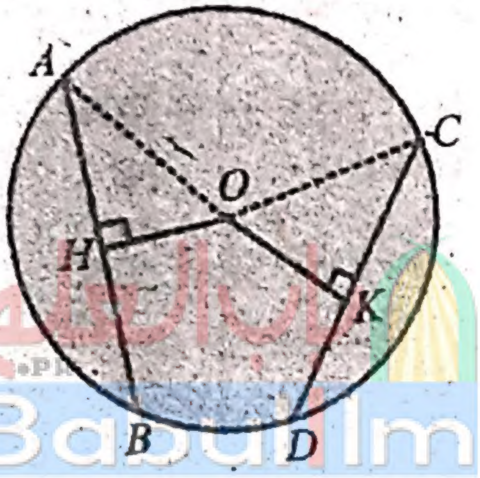


Steps:

1. Draw a diameter AD = 6 cm.
2. From point A draw an arc of radius $\overline{AO}$ = 3 cm, which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U, respectively.
6. Thus PQRSTU is the circumscribed regular hexagon.

Q.9. Prove that if two chords of a circle are congruent, then they will be equidistant from the centre. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C.

So that we have ∠rtΔ^s OAH and OCK.

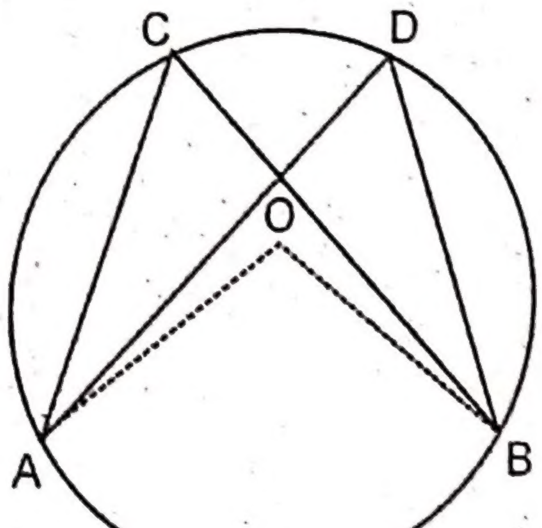
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly, $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle$rt Δ^s OAH $\leftrightarrow$ OCK	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

OR

Prove that any two angles in the same segment of a circle are equal.

Ans



Given:

$m\angle ACB = m\angle ADB$ are the circumangles in the same segment of a circle with centre O.

To Prove:

$m\angle ACB = m\angle ADB$

Construction:

Join O with A and O with B.

So that $\angle AOB$ is the central angle.

Proof:

Statements	**Reasons**
Standing on the same arc AB of a circle.	
$\angle AOB$ is the central angle whereas	Construction
$\angle ACB$ and $\angle ADB$ are circumangles	Given
$\therefore\ m\angle AOB = 2m\angle ACB$ (i)	By theorem I (External angle is the sum internal opposite angle).
and $m\angle AOB = 2m\angle ADB$ (ii)	By theorem I
$\Rightarrow\ 2m\angle ACB = 2m\angle ADB$	Using (i) and (ii)
Hence, $m\angle ACB = m\angle ADB$	